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We are interested in three very important quantities:

1. the optimal savings rate $s_j = \frac{w_j}{c_j}$ during your working years;
2. the optimal trajectory of financial capital $F_j$ over your lifecycle, and;
3. the optimal retirement spending rate $c_j / (F_j + c_j)$ once your wage income is zero, i.e. $j > R$ (more on this in later lectures.)

These three values will depend on your personal patience rate, denoted by $k$, your retirement horizon $R$ (in years) and the overall length of life $D$ (in years.)
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These three values will depend on your personal patience rate, denoted by \( k \), your retirement horizon \( R \) (in years) and the overall length of life \( D \) (in years.)
Remember the Timelines

- The value of human capital at time zero is the present value of wages until retirement:

\[ H_0 = \sum_{j=1}^{R} \frac{w_j}{(1 + v)^j} = w_0 \sum_{j=1}^{R} \frac{(1 + g_w)^j}{(1 + v)^j} = w_0 \cdot PVA(g_w, v, R). \]

Remember that the first payment is at end of year \( j = 1 \). The last payment is at end of year \( j = R \).
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Remember that the first payment is at end of year \( j = 1 \). The last payment is at end of year \( j = R \).

- Pay careful attention to the evolution of financial capital:

\[
\begin{align*}
F_1 &= F_0 (1 + v) + w_1 - c_1 \\
F_2 &= F_1 (1 + v) + w_2 - c_2 \\
F_R &= F_{R-1} (1 + v) + w_R - c_R \\
F_{R+1} &= F_R (1 + v) - c_{R+1}.
\end{align*}
\]
Interest (valuation) rates are $v = 3\%$ in real (inflation adjusted) terms. Make sure you understand what this means.
Question #1a:

- Interest (valuation) rates are $v = 3\%$ in real (inflation adjusted) terms. Make sure you understand what this means.

- You are just about to turn 25 (tomorrow) and have just received your annual paycheck of $w_0 = 50,000$ which is expected to grow at a real rate of $g_w = 1\%$ per year.
Question #1a:

- Interest (valuation) rates are $v = 3\%$ in real (inflation adjusted) terms. Make sure you understand what this means.

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- You spent your entire paycheck on a wild birthday party, and woke up (hung over) the next morning (broke) realizing that you are about to turn 25 and should start planning your financial future.
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- You spent your entire paycheck on a wild birthday party, and woke up (hung over) the next morning (broke) realizing that you are about to turn 25 and should start planning your financial future.

- After careful thought you have determined that you would like to enjoy a constant real standard of living for the rest of your life, which you estimate to be: $(90 - 25) = 65$ years.
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- Interest (valuation) rates are \( v = 3\% \) in real (inflation adjusted) terms. Make sure you understand what this means.
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- You spent your entire paycheck on a wild birthday party, and woke up (hung over) the next morning (broke) realizing that you are about to turn 25 and should start planning your financial future.
- After careful thought you have determined that you would like to enjoy a constant real standard of living for the rest of your life, which you estimate to be: \( (90 - 25) = 65 \) years.
- **Question:** What is your optimal consumption amount \( c_1^* \) and optimal savings amount \( s_1^* \) at the end of the first year of savings?
**Answer:** The value of your human capital (today) is:

$$H_0 = 50,000 \cdot PVA(0.01, 0.03, 40) = 50,000(27.45072) = \$1,372,536$$
Answer: The value of your human capital (today) is:

\[ H_0 = 50,000 \cdot PVA(0.01, 0.03, 40) = 50,000(27.45072) = $1,372,536 \]

You have no financial capital, so your economic net worth (ENW) is \( W_0 = $1,372,536 \) as well.
Question #1a: (solved)

- **Answer:** The value of your human capital (today) is:

\[ H_0 = 50,000 \cdot PVA(0.01, 0.03, 40) = 50,000(27.45072) = 1,372,536 \]

- You have no financial capital, so your economic net worth (ENW) is \( W_0 = 1,372,536 \) as well.

- You would like to spend \( W_0 \) evenly over the next 65 years of life, so the optimal baseline consumption rate is:

\[ c_0^* = \frac{1,372,536}{PVA(0.0, 0.03, 65)} = \frac{1,372,536}{28.45289} = 48,239. \]
**Question #1a: (solved)**

- **Answer**: The value of your human capital (today) is:
  \[ H_0 = 50,000 \cdot PVA(0.01, 0.03, 40) = 50,000(27.45072) = $1,372,536 \]

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  \[
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  \]

- This leads to an optimal savings rate at (year end) of:
  \[
  s_1^* = \frac{50,000(1 + 0.01) - 48,239(1 + 0.0)}{50,000(1 + 0.01)} = \frac{2,261}{50,500} = 4.48\%
  \]
Question #1a: (solved)

- **Answer:** The value of your human capital (today) is:
  \[ H_0 = 50,000 \cdot PVA(0.01, 0.03, 40) = 50,000 \times (27.45072) = $1,372,536 \]

- You have no financial capital, so your economic net worth (ENW) is \( W_0 = $1,372,536 \) as well.

- You would like to spend \( W_0 \) evenly over the next 65 years of life, so the optimal baseline consumption rate is:
  \[ c^*_0 = \frac{1,372,536}{PVA(0.0, 0.03, 65)} = \frac{1,372,536}{28.45289} = $48,239. \]

- This leads to an optimal savings rate at (year end) of:
  \[ s^*_1 = \frac{50,000(1 + 0.01) - 48,239(1 + 0.0)}{50,000(1 + 0.01)} = \frac{2,261}{50,500} = 4.48\% \]

- So, at the end of your 25th year of life, just before your 26th birthday, make sure to save $2,261 and enjoy the rest.
Question #1b:

**Question:** How much financial capital ($F_{10}^*$) will you accumulate over 10 years, just as you are about to turn 35?
Question #1b:

- **Question:** How much financial capital \( F_{10}^* \) will you accumulate over 10 years, just as you are about to turn 35?
- **HINT:** Remember the fundamental identity:

\[
F_j^* + H_j = c_j^* \cdot \text{PVA}(k, v, D - j)
\]
Question #1b:

- **Question**: How much financial capital ($F_{10}^*$) will you accumulate over 10 years, just as you are about to turn 35?

- **HINT**: Remember the fundamental identity:

  $$F_j^* + H_j = c_j^* \cdot \text{PVA}(k, \nu, D - j)$$

- **Answer**: Compute the present value of human capital at time $j$,

  \[
  H_j = w_0(1 + g_w)^j \cdot \text{PVA}(g_w, \nu, R - j)
  \]

  \[
  H_{10} = 50,000(1.01)^{10} \cdot \text{PVA}(0.01, 0.03, 30)
  = (55231.11)(22.457557) = $1,240,356
  \]
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$$= (55231.11)(22.457557) = $1,240,356$$

The present value of optimal (remaining) lifetime consumption is:

$$c_0^*(1 + k)^j \cdot PVA(k, v, D - j) = (48,238.9)(26.77443) = $1,291,569$$
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The optimal value of financial capital at time $j = 10$ is:
  \[ F_{10}^* = 1,291,569 - 1,240,356 = $51,213 \]
Case #1: Wages, Consumption, Financial Capital and Human Capital: Wage growth of $g=1\%$, personal patience of $k=0\%$ and valuation $v=3\%$.
Case #1: Wages, Consumption, Financial Capital and Human Capital: Wage growth of g=1%, personal patience of k=0% and valuation v=3%
Question: How much financial capital \( F_{40}^* \) will you accumulate over \( j = 40 \) years, just as you are about to turn 65?

HINT: Remember the fundamental identity:

\[
F_{j+1} + H_j = c_j PVA(k, v, D_j)
\]

Answer: Compute the present value of human capital.

\[
H_{40} = \$0
\]

The present value of optimal (remaining) lifetime consumption is:

\[
c_0(1+k)^j PVA(k, v, D_j) = (48,238.9)(17.41315) = \$839,991
\]

The optimal value of financial capital at time \( j = 40 \) is (by definition):

\[
F_{40}^* = \$839,991
\]

This is often called your retirement nest egg.
Question #1c

- **Question**: How much financial capital \( F_{40}^* \) will you accumulate over \( j = 40 \) years, just as you are about to turn 65?

- **HINT**: Remember the fundamental identity:

\[
F_j^* + H_j = c_j^* \cdot \text{PVA}(k, \nu, D - j)
\]

Answer:

Compute the present value of human capital. \( H_{40} = 0 \)

The present value of optimal (remaining) lifetime consumption is:

\[
c_0 \left(1 + k\right)^j \cdot \text{PVA}(k, \nu, D - j) = (48,238.9)(17.41315) = 839,991
\]

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- **Question**: How much financial capital \( F_{40}^* \) will you accumulate over \( j = 40 \) years, just as you are about to turn 65?

- **HINT**: Remember the fundamental identity:

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\]

- **Answer**: Compute the present value of human capital.

\[
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  \]
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- **Question:** How much financial capital ($F^*_{40}$) will you accumulate over $j = 40$ years, just as you are about to turn 65?
- **HINT:** Remember the fundamental identity:

  $$F_j^* + H_j = c_j^* \cdot \text{PVA}(k, v, D - j)$$

- **Answer:** Compute the present value of human capital.

  $$H_{40} = \$0$$

  The present value of optimal (remaining) lifetime consumption is:

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  The optimal value of financial capital at time $j = 40$ is (by definition):

  $$F^*_{40} = \$839,991$$

  This is often called your retirement nest egg.
Summary Values Question #1

Summary values for wages, optimal consumption, optimal savings amount, optimal savings rate and financial capital, assuming $v = 3\%$ and $k = 0\%$.

<table>
<thead>
<tr>
<th>Year #</th>
<th>Wage</th>
<th>Consume</th>
<th>Saving</th>
<th>Rate</th>
<th>Fin. Cap.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j$</td>
<td>$w_j$</td>
<td>$c_j^*$</td>
<td>$s_j^*$</td>
<td>$s_j^*/w_j$</td>
<td>$F_j^*$</td>
</tr>
<tr>
<td>0</td>
<td>$50,000$</td>
<td>$48,239$</td>
<td>$1,761$</td>
<td>3.52%</td>
<td>$0$</td>
</tr>
<tr>
<td>1</td>
<td>$50,500$</td>
<td>$48,239$</td>
<td>$2,261$</td>
<td>4.48%</td>
<td>$2,261$</td>
</tr>
<tr>
<td>10</td>
<td>$55,231$</td>
<td>$48,239$</td>
<td>$6,992$</td>
<td>12.66%</td>
<td>$51,213$</td>
</tr>
<tr>
<td>25</td>
<td>$64,122$</td>
<td>$48,239$</td>
<td>$15,883$</td>
<td>24.77%</td>
<td>$289,894$</td>
</tr>
<tr>
<td>40</td>
<td>$74,443$</td>
<td>$48,239$</td>
<td>$26,204$</td>
<td>35.20%</td>
<td>$839,991$</td>
</tr>
<tr>
<td>41</td>
<td>$0$</td>
<td>$48,239$</td>
<td>$0$</td>
<td>N.A.</td>
<td>$816,952$</td>
</tr>
<tr>
<td>65</td>
<td>$0$</td>
<td>$48,239$</td>
<td>$0$</td>
<td>N.A.</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Note: Time zero are baseline consumption amounts.
Question #2:

Question: Assume that instead of $k = 0$, that $k = 1\%$ and you would like your standard of living to increase (smoothly) by 1% per year. What is the optimal baseline consumption rate?
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Answer: The value of your human capital (today) and ENW is still $1,372,536$
Question #2:

- Question: Assume that instead of $k = 0$, that $k = 1\%$ and you would like your standard of living to increase (smoothly) by 1% per year. What is the optimal baseline consumption rate?
- **Answer:** The value of your human capital (today) and ENW is still $1,372,536$
- You would like to spend $H_0$ evenly over the next 65 years of life, so the optimal baseline consumption rate:

\[
c_0^* = \frac{1,372,536}{PVA(0.01, 0.03, 65)} = $37,725
\]

which is obviously lower than the previous $48,239$. Why?
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which is obviously lower than the previous $48,239$. Why?

This leads to an optimal savings rate at (year end) of:

$$s_1^* = \frac{50,000(1.01) - 37,725(1.01)}{50,000(1.01)} = \frac{50000 - 37725}{50000} = 24.55\%$$
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  \]

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  \[
  s_1^* = \frac{50,000(1.01) - 37,725(1.01)}{50,000(1.01)} = \frac{50000 - 37725}{50000} = 24.55\%
  \]

- Notice that the savings rate does not depend on time (or age.)
Case #2: Wages, Consumption, Financial Capital and Human Capital: Wage growth of $g=1\%$, personal patience of $k=1\%$ and valuation $v=3\%$
Case #2: Wages, Consumption, Financial Capital and Human Capital: Wage growth of $g=1\%$, personal patience of $k=1\%$ and valuation $v=3\%$
Summary values for wages, optimal consumption, optimal savings amount, optimal savings rate and financial capital, assuming $v = 3\%$ and $k = 1\%$.

<table>
<thead>
<tr>
<th>Year #</th>
<th>Wage</th>
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<td>$j$</td>
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<td>$F_j^*$</td>
</tr>
<tr>
<td>0</td>
<td>$50,000$</td>
<td>$37,725$</td>
<td>$12,275$</td>
<td>24.55%</td>
<td>$0$</td>
</tr>
<tr>
<td>1</td>
<td>$50,500$</td>
<td>$38,103$</td>
<td>$12,397$</td>
<td>24.55%</td>
<td>$12,397$</td>
</tr>
<tr>
<td>10</td>
<td>$55,231$</td>
<td>$41,672$</td>
<td>$13,559$</td>
<td>24.55%</td>
<td>$148,332$</td>
</tr>
<tr>
<td>25</td>
<td>$64,122$</td>
<td>$48,380$</td>
<td>$15,741$</td>
<td>24.55%</td>
<td>$502,932$</td>
</tr>
<tr>
<td>40</td>
<td>$74,443$</td>
<td>$56,168$</td>
<td>$18,275$</td>
<td>24.55%</td>
<td>$1,099,143$</td>
</tr>
<tr>
<td>41</td>
<td>$0$</td>
<td>$56,729$</td>
<td>$0$</td>
<td>N.A.</td>
<td>$1,075,388$</td>
</tr>
<tr>
<td>65</td>
<td>$0$</td>
<td>$72,031$</td>
<td>$0$</td>
<td>N.A.</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Note: Time zero are baseline consumption amounts.
Question #3:

Question: Now assume that $k = -1\%$ and you would like your standard of living to start very high and then decline (smoothly) by 1% per year. What is the optimal initial consumption rate?

Answer: Value of your human capital is still $1,372,536. You would like to spend $H_0$ evenly over the next 65 years of life, so the baseline consumption rate is:

$$c_0 = 1,372,536 \text{PVA}(0.01, 0.03, 65) = 60,029,029,$$

which is obviously much higher than your baseline wage of $w_0 = 50,000$. This leads to a negative savings rate at (year end) of:

$$s_1 = 50,000 (1.01) 60,029 (0.99) 50,500 = 8,929,500,$$

17.68%. You will spend almost 18% more than what you make, by taking on debt.
Question #3:

- **Question:** Now assume that $k = -1\%$ and you would like your standard of living to start very high and then decline (smoothly) by 1% per year. What is the optimal initial consumption rate?
- **Answer:** Value of your human capital is still $1,372,536$

You would like to spend $H_0$ evenly over the next 65 years of life, so the baseline consumption rate is:

$$c_0 = \frac{1,372,536}{60,029} \approx 22.84$$

which is obviously much higher than your baseline wage of $w_0 = 50,000$

This leads to a negative savings rate at (year end) of:

$$s_1 = 50,000 \cdot (1.01) \cdot 60,029 \cdot (0.99) \cdot 50,000 \cdot (1.01) \approx -8,929$$

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- **Answer:** Value of your human capital is still $1,372,536
- You would like to spend \( H_0 \) evenly over the next 65 years of life, so the baseline consumption rate is:

\[
c_0^* = \frac{1,372,536}{PVA(-0.01, 0.03, 65)} = $60,029,
\]

which is obviously much higher than your baseline wage of \( w_0 = $50,000 \).
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- Question: Now assume that \( k = -1\% \) and you would like your standard of living to start very high and then decline (smoothly) by 1\% per year. What is the optimal initial consumption rate?

- Answer: Value of your human capital is still $1,372,536

You would like to spend \( H_0 \) evenly over the next 65 years of life, so the baseline consumption rate is:

\[
c_0^* = \frac{1,372,536}{PVA(-0.01, 0.03, 65)} = $60,029,
\]

which is obviously much higher than your baseline wage of \( w_0 = $50,000 \).

- This leads to a negative savings rate at (year end) of:

\[
s_1^* = \frac{50,000(1.01) - 60,029(0.99)}{50,000(1.01)} = \frac{-8,929}{50,500} = -17.68\%\]
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- You will spend almost 18% more than what you make, by taking on debt.
When Will You Start Saving?

**Question:** When will the optimal savings rate be positive?

Answer: You must solve the following equation:

\[ s_j = w_0 \left(1 + g w_j c_0 \right) - k_0 \left(1 + g w_j \right) = 0. \]

This leads to:

\[ 1 + k_1 + g w_j = w_0 c_0. \]

In our case it results in:

\[ j \ln 0.99101 = \ln 50000 \times 0.0029. \]

So that:

\[ j = \frac{\ln 50000 \times 0.0029}{\ln 0.99101} = 9.139, \]

and in the 10th year (at age 35) the savings rate is positive for the first time.
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\end{align*}
\]

This leads to:

\[
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\end{align*}
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In our case it results in:

\[ j \ln \left[ \frac{0.99}{1.01} \right] = \ln \left[ \frac{50000}{60029} \right] \]

so that:

\[ j = \frac{\ln \left[ \frac{50000}{60029} \right]}{\ln \left[ \frac{0.99}{1.01} \right]} \approx 9.139, \]

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So that:

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Case #3: Wages, Consumption, Financial Capital and Human Capital: Wage growth of $g=1\%$, personal patience of $k=-1\%$ and valuation $v=3\%$
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Wage growth of $g=1\%$, personal patience of $k=-1\%$ and valuation $v=3\%$
Summary Values Question #3

Summary values for wages, optimal consumption, optimal savings amount, optimal savings rate and financial capital, assuming $\nu = 3\%$ and $k = -1\%$.

<table>
<thead>
<tr>
<th>Year #</th>
<th>Wage</th>
<th>Consume</th>
<th>Saving</th>
<th>Rate</th>
<th>Fin. Cap.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j$</td>
<td>$w_j$</td>
<td>$c_j^*$</td>
<td>$s_j^*$</td>
<td>$s_j^*/w_j$</td>
<td>$F_j^*$</td>
</tr>
<tr>
<td>0</td>
<td>$50,000$</td>
<td>$60,029$</td>
<td>$-10,029$</td>
<td>$-20.06%$</td>
<td>$0$</td>
</tr>
<tr>
<td>1</td>
<td>$50,500$</td>
<td>$59,429$</td>
<td>$-8,929$</td>
<td>$-17.68%$</td>
<td>$-8,929$</td>
</tr>
<tr>
<td>10</td>
<td>$55,231$</td>
<td>$54,289$</td>
<td>$942$</td>
<td>$1.7%$</td>
<td>$-48,809$</td>
</tr>
<tr>
<td>25</td>
<td>$64,122$</td>
<td>$46,692$</td>
<td>$17,430$</td>
<td>$27.18%$</td>
<td>$93,498$</td>
</tr>
<tr>
<td>40</td>
<td>$74,443$</td>
<td>$40,158$</td>
<td>$34,285$</td>
<td>$46.06%$</td>
<td>$624,680$</td>
</tr>
<tr>
<td>41</td>
<td>$0$</td>
<td>$39,756$</td>
<td>$0$</td>
<td>N.A.</td>
<td>$603,664$</td>
</tr>
<tr>
<td>64</td>
<td>$0$</td>
<td>$31,236$</td>
<td>$0$</td>
<td>N.A.</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Note: Time zero are hypothetical (baseline) amounts.
Here are some qualitative facts about consumption and savings rate, given the same $H_0$ and $v$ (as well as $R$ and $D$).

- Tell me your $k$ and I’ll show you your $c_0^*$. 

Take-aways

Here are some qualitative facts about consumption and savings rate, given the same $H_0$ and $\nu$ (as well as $R$ and $D$.)

- Tell me your $k$ and I’ll show you your $c_0^*$.  
- Show me your $c_0^*$ and I can figure-out your $k$.  

Impatient consumers borrow money and live in debt early in life.  They will accumulate less wealth at retirement.  The must save a higher percentage of their final-years salary.  They obviously are taking on some risk...

CHM (Cambridge 2012)  
Strategic FP over L  
Ch. #4: Lecture Notes
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