

Strategic Financial Planning over the Lifecycle

Chapter #4: Consumption Smoothing

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Ch. #4: Lecture Notes

Background

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- These three values will depend on your personal patience rate, denoted by k , your retirement horizon R (in years) and the overall length of life D (in years.)

Remember the Timelines

- The value of human capital at time zero is the present value of wages until retirement:

$$H_0 = \sum_{j=1}^R \frac{w_j}{(1+v)^j} = w_0 \sum_{j=1}^R \frac{(1+g_w)^j}{(1+v)^j} = w_0 \cdot \mathbf{PVA}(g_w, v, R).$$

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- Pay careful attention to the evolution of financial capital:

$$F_1 = F_0(1+v) + w_1 - c_1$$

$$F_2 = F_1(1+v) + w_2 - c_2$$

$$F_R = F_{R-1}(1+v) + w_R - c_R$$

$$F_{R+1} = F_R(1+v) - c_{R+1}.$$

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- After careful thought you have determined that you would like to enjoy a constant real standard of living for the rest of your life, which you estimate to be: $(90 - 25) = 65$ years.
- **Question:** What is your optimal consumption amount c_1^* and optimal savings amount s_1^* at the end of the first year of savings?

Question #1a: (solved)

- **Answer:** The value of your human capital (today) is:

$$H_0 = 50,000 \cdot \mathbf{PVA}(0.01, 0.03, 40) = 50,000(27.45072) = \$1,372,536$$

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- So, at the end of your 25th year of life, just before your 26th birthday, make sure to save \$2,261 and enjoy the rest.

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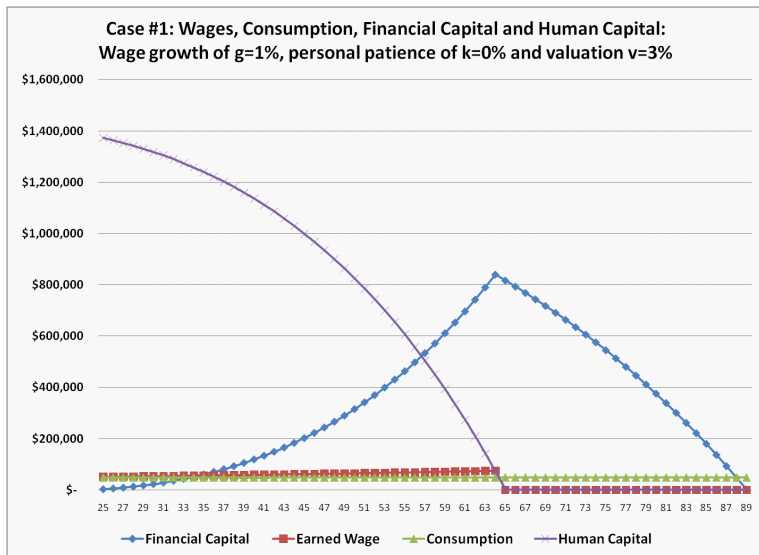
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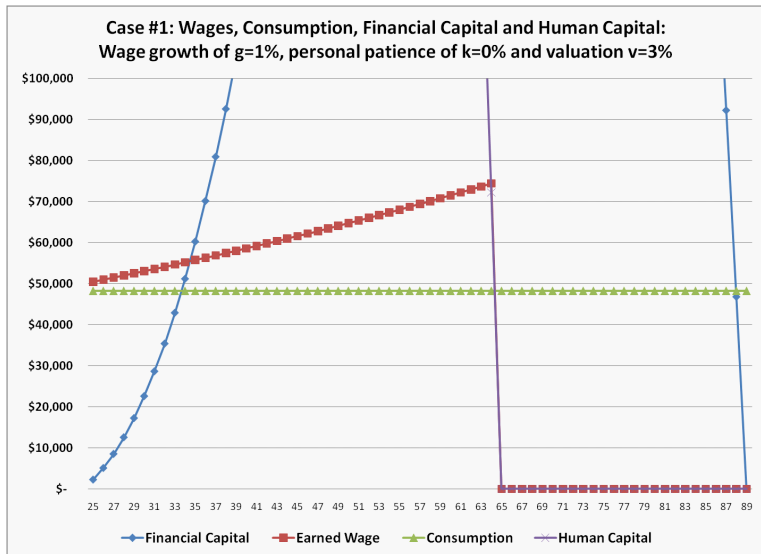
- The optimal value of financial capital at time $j = 10$ is:

$$F_{10}^* = 1,291,569 - 1,240,356 = \$51,213$$

LifeCycle Model: Case #1



LifeCycle Model: Case #1 (zoom)



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- This is often called your retirement nest egg.

Summary Values Question #1

Summary values for wages, optimal consumption, optimal savings amount, optimal savings rate and financial capital, assuming $v = 3\%$ and $k = 0\%$.

Year #	Wage	Consume	Saving	Rate	Fin. Cap.
j	w_j	c_j^*	s_j^*	s_j^* / w_j	F_j^*
0	\$50,000	\$48,239	\$1,761	3.52%	\$0
1	\$50,500	\$48,239	\$2,261	4.48%	\$2,261
10	\$55,231	\$48,239	\$6,992	12.66%	\$51,213
25	\$64,122	\$48,239	\$15,883	24.77%	\$289,894
40	\$74,443	\$48,239	\$26,204	35.20%	\$839,991
41	\$0	\$48,239	\$0	N.A.	\$816,952
65	\$0	\$48,239	\$0	N.A.	\$0
Note: Time zero are baseline consumption amounts.					

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which is obviously lower than the previous \$48,239. Why?

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- This leads to an optimal savings rate at (year end) of:

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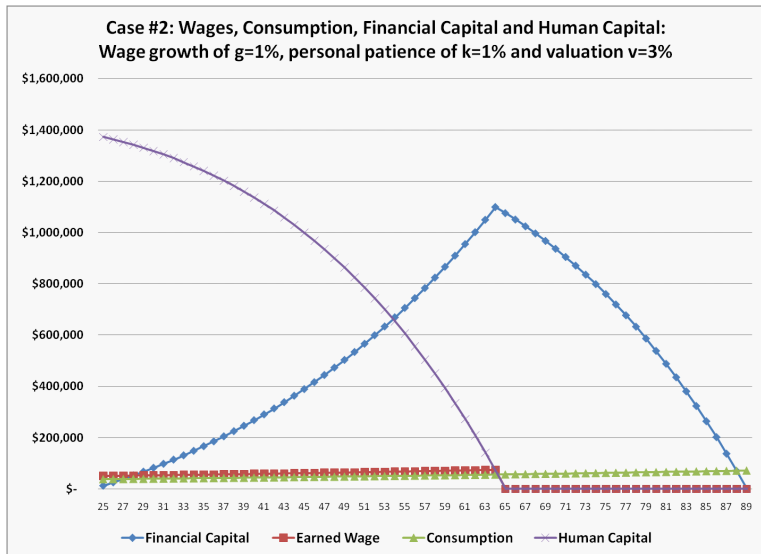
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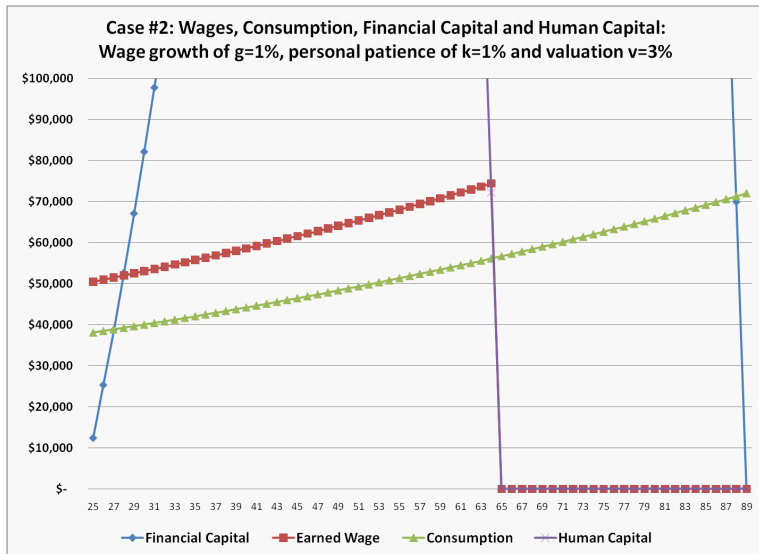
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- Notice that the savings rate does not depend on time (or age.)

LifeCycle Model: Case #2



LifeCycle Model: Case #2 (zoom)



Summary Values Question #2

Summary values for wages, optimal consumption, optimal savings amount, optimal savings rate and financial capital, assuming $v = 3\%$ and $k = 1\%$.

Year #	Wage	Consume	Saving	Rate	Fin. Cap.
j	w_j	c_j^*	s_j^*	s_j^* / w_j	F_j^*
0	\$50,000	\$37,725	\$12,275	24.55%	\$0
1	\$50,500	\$38,103	\$12,397	24.55%	\$12,397
10	\$55,231	\$41,672	\$13,559	24.55%	\$148,332
25	\$64,122	\$48,380	\$15,741	24.55%	\$502,932
40	\$74,443	\$56,168	\$18,275	24.55%	\$1,099,143
41	\$0	\$56,729	\$0	N.A.	\$1,075,388
65	\$0	\$72,031	\$0	N.A.	\$0
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- You would like to spend H_0 evenly over the next 65 years of life, so the baseline consumption rate is:

$$c_0^* = \frac{1,372,536}{\text{PVA}(-0.01, 0.03, 65)} = \$60,029,$$

which is obviously much higher than your baseline wage of $w_0 = \$50,000$.

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- You will spend almost 18% more than what you make, by taking on debt.

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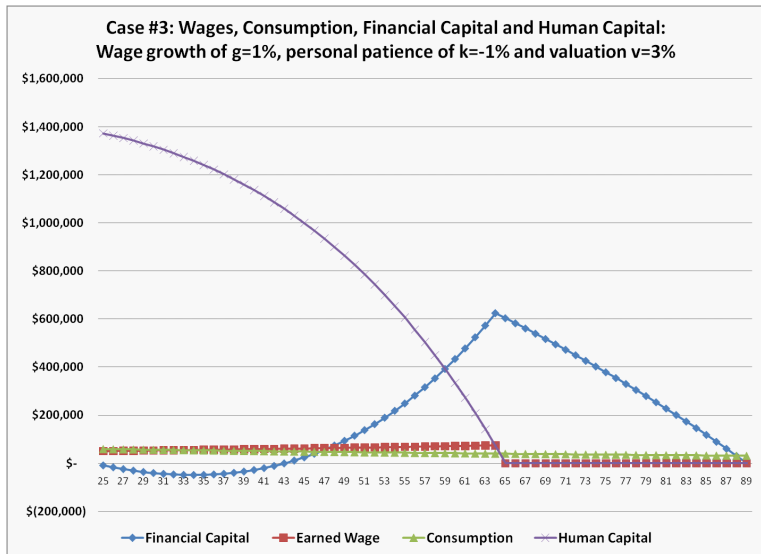
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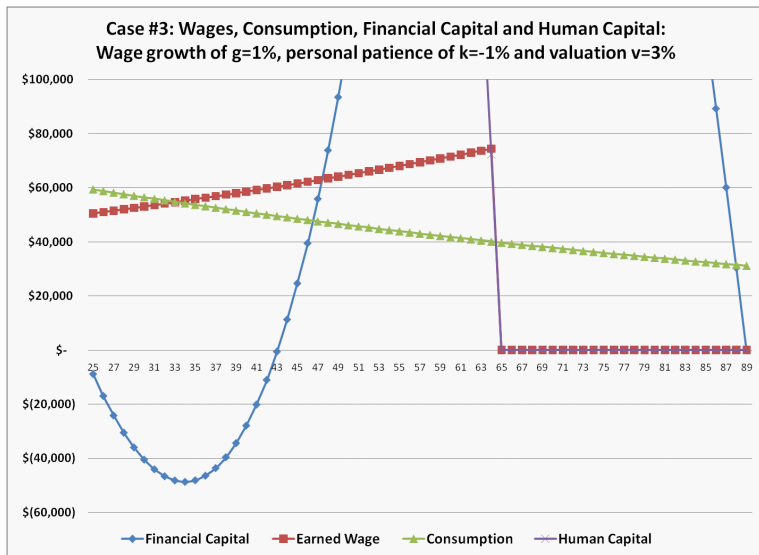
$$j = \ln \left[\frac{50000}{60029} \right] / \ln \left[\frac{0.99}{1.01} \right] = 9.139,$$

and in the 10th year (at age 35) the savings rate is positive for the first time.

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25	\$64,122	\$46,692	\$17,430	27.18%	\$93,498
40	\$74,443	\$40,158	\$34,285	46.06%	\$624,680
41	\$0	\$39,756	\$0	N.A.	\$603,664
64	\$0	\$31,236	\$0	N.A.	\$0

Note: Time zero are hypothetical (baseline) amounts.

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- They obviously are taking on some risk...