Strategic Financial Planning over the Lifecycle

Chapter #8: Mortality Risk and Insurance

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Ch. #8: Lecture Notes

Learning Objectives

- Who needs life insurance (LI), why, and how much do they need?

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- What does it cost and how is LI valued and priced?
- What are the tax implications of buying and selling a LI policy?
- What are the three (3) major types of life insurance?
- Finally: Is selling life insurance a good way to make a living?

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- His doctor has warned Joe that if he isn't careful, he might get a heart-attack that will be fatal.
- QUESTION: Given his age and the dire medical warnings, should Joe consider buying some life insurance?

More Information About Joe

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- **Question**: Does Joe really need (more) life insurance?

Question: Does Joanne Need Life Insurance?

- Joanne is 32 years old, with three lovely children ages 2, 5 and 8. She is working as a professor of Viking literature at a large Canadian university. Her common-law partner Frank is 28 and stays home to raise the kids. In his spare time he is working on (writing) a great novel and does some freelance magazine editing to pay-off student loans.

Question: Does Joanne Need Life Insurance?

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- **Question #1**: Do they need (more) Life Insurance?
- **Question #2**: Do they need any other type of insurance?

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- Life insurance is sold, not bought.

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- These insurance companies also sell pension annuities and health-benefit plans, for total premium income of **$80** billion in 2009.

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- Life insurance enjoys a very special (beneficial) tax treatment compared to other investment vehicles and the industry lobbies very aggressively to maintain this favorable treatment.

What Does Basic Life Insurance Cost?

- The single most important factor is your age/gender.

Monthly Premiums for $100K of Term LI (Excellent Health)						
Term of	Age 30		Age 50		Age 70	
Insurance	*Male*	*Fem.*	*Male*	*Fem.*	*Male*	*Fem.*
5 years	12.71	11.53	19.65	15.30	105.65	59.27
10 years	8.21	7.68	17.95	14.57	102.51	55.96
20 years	11.01	9.68	27.56	21.19	207.54	128.07
30 years	15.47	12.88	46.23	33.15	307.33	259.50
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- Review the relationship with each-other carefully. Do they make sense?

Impact of Health Status on LI Cost

Monthly Premiums for $100,000 of Term Life Insurance						
50 year-old with varying Health Status (Scale of 1 to 5)						
	Average (2/5)		Excellent (4/5)		Exceptional (5/5)	
Term	*Male*	*Fem.*	*Male*	*Fem.*	*Male*	*Fem.*
5 yrs	27.61	20.68	19.65	15.30	15.37	12.11
10 yrs	23.54	18.38	17.95	14.57	14.86	12.48
20 yrs	38.69	28.65	27.56	21.19	23.85	17.90

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- In particular, (1.) interest rates and (2.) mortality rates (i.e. your age, gender and health) play a big role in pricing.
- Long-term rates are currently at historical lows. What impact do you think this has on insurance prices?

THE Insurance & Investment JOURNAL

LSM Insurance was featured in February issue of [The Insurance Journal](). Enjoy the article!

Universal life:

Low interest rates drive up prices

Low interest rates have diminished the profitability of level cost universal life insurance for years. Although fierce competition may have overruled common sense for some time, manufacturers no longer have a choice. Long-term interest rates have finally forced changes.

Based on outdated actuarial assumptions, level cost universal life insurance needed a serious overhaul to return to profitability. After bottoming out in 2008, long-term interest rates once again fell below 3.5% just before year

Alliance boosted its level cost on Jan.17. **Sun Life Financial** followed on Jan. 28 and **Canada Life** did the same on Feb. 7. **Empire Life** announced that it would make a similar

Alliance, told *The Insurance and Investment Journal*. With this generalized increase, he thinks the industry has solved its interest rate problem for 2011.

Sun Life explains the increase to its advisors on its website: "Interest rates have continued to fall, with long-term Canadian bond yields falling to their lowest levels in recent history...This continued low interest rate environment puts pressure on margins for

Introduction to Actuarial Mortality Rates and Tables

Canadian Population 2000/2002		
Annual Death (Mortality) Rate		
Probability of Death $\times$ 1000		
Age	**Female**	**Male**
20	0.340	0.820
30	0.390	0.880
40	0.920	1.520
50	2.290	3.600
60	5.870	9.820
70	14.930	25.550
80	42.400	68.460
90	130.880	182.640
Statistics Canada (84-537-XIE)		

Actuarial Mortality Rates: Part II

- A mortality table maps an age (x) into a probability of death q_x, during the next year. By definition, $0 \leq q_x \leq 1$ and $q_N = 1$, for some large enough $N \approx 110$.

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- Mind your p's and q's! It can get confusing.

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- Remember: Only Four (4) things can happen.

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which is $1.00 - 0.99574 = 0.4253\%$ for males and $1.00 - 0.99893 = 0.1679\%$ for females. Note that you do *not* add-up the age-dependent death rates, which are conditional probabilities.

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- **Answer**:

$$1 - (0.998321)(0.995747) = 0.59245\%$$

Numerical Examples: Mortality

- **Question**: Using Canadian population mortality rates, what is the probability that a 20-year old (male/female) dies within the next five years, before the age of 25?
- **Answer:** Multiply the following together:

$$1 - (1 - q_{20})(1 - q_{21})(1 - q_{22})(1 - q_{23})(1 - q_{24}),$$

  which is $1.00 - 0.99574 = 0.4253\%$ for males and $1.00 - 0.99893 = 0.1679\%$ for females. Note that you do *not* add-up the age-dependent death rates, which are conditional probabilities.
- **Question:** What is the probability that at-least one member of a 20-year old couple (male/female) dies before their 25th birthday?
- **Answer**:

$$1 - (0.998321)(0.995747) = 0.59245\%$$

- Are these probabilities independent, in practice?

Life Expectancy: The 50% Mark

Life Expectancy at Birth in 2005	
Bottom 10 Countries	**Top 10 Countries**
Swaziland (35.3)	Japan (82.4)
Lesotho (36.3)	Sweden (80.7)
Djibouti (37.6)	Hong Kong (80.6)
Botswana (38.2)	Macao (80.07)
Mozambique (38.4)	Israel (79.97)
Malawi (40.52)	Iceland (79.91)
Sierra Leone (42.37)	Norway (79.73)
South Africa (42.44)	France (79.69)
Burundi (42.66)	Australia (79.64)
Rwanda (43.33)	Belgium (79.59)
Source: Watson Wyatt	

Fair Actuarial Premium: One Year Term

- Consider a one-year term life insurance policy for an x-year-old individual, paying \$1 (at the end of the year) if the insured dies at any time during the year. The mortality rate is q_x, which implies that if the insurance company sells N of these polices it will have to pay death benefit claims on approximately $q_x N$ policies. The $q_x N$ will be paid at the end of the year, so its present value is: $q_x N/(1+v)$, where v denotes the valuation rate.

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- If the company is to insure N lives, then it must collect $q_x N/(1+v)$ up-front from the group of N people. This implies that it must collect $q_x/(1+v)$ per policy holder, under the assumption the company is non-profit enterprise.

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- The **Actuarial Premium** of a one-year life insurance policy is:

$$A_{x:1} := \frac{q_x}{1+v},$$

Actuarial Premium vs. Insurance Cost

- The quantity $A_{x:1}$ is often referred to as the **actuarial** net single premium (NSP). The word actuarial is meant to remind you that the only thing the premium covers, is the pure death benefit. It does not account for profits or anything non actuarial.

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- In practice, of course, you would have to pay more than the no-profit (actuarial) cost to the insurance company, and so with a slight play on words, the **Insurance Cost** is defined as:

$$\textbf{Insurance Cost} = (1 + \Lambda) \times \textbf{Actuarial Premium}$$

where the symbol $\Lambda > 0$ denotes the percentage profit plus commission plus fees (loading) above and beyond the pure actuarial cost of insurance.

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- What would this look like for 3 years of coverage?

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- Note that rarely if ever do people pay the entire $A_{x:n}$ up-front and in one lump-sum. It is often amortized over the n-year period, which is actually more complicated than you think...

Numerical Examples: Insurance

- **Question**: Using the Canadian population mortality rates, please compute the **monthly** actuarial cost/price of a 1-year term life insurance policy for a Canadian male/female that pays \$100,000 if the individual dies during the year. Do this for age 30, 50 and then 70, assuming a $v = 5\%$ valuation rate.

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- How does this compare to the real-world prices I showed earlier? Why the discrepancy?

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- Think of any insurance policy as money going into a (big) lock-box with two compartments. Some of the money is dedicated to pure protection, and other to pure investment. Each year you can decide how to move money between the two compartments.

Summary Table: 3 Different Life Insurance Policies

Types of Life Insurance (in Canada)			
	Term	**Whole Life**	**Universal Life**
Features / Category	*(Temporary)*	*(Permanent)*	
Tax-sheltered Savings:			
Regular Dividends:			
Investment Options:			
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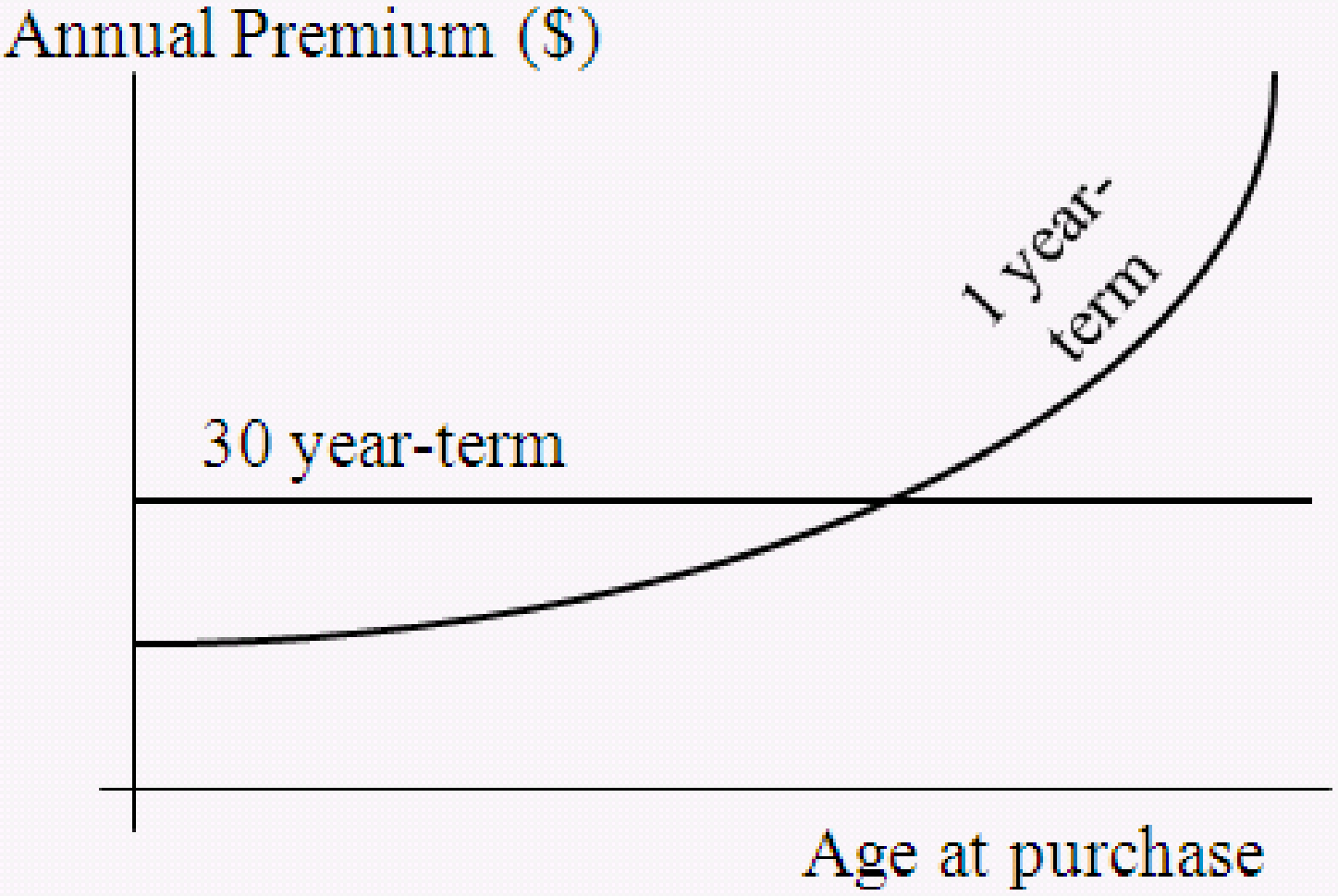
Tradeoff: Short Term vs. Long Term



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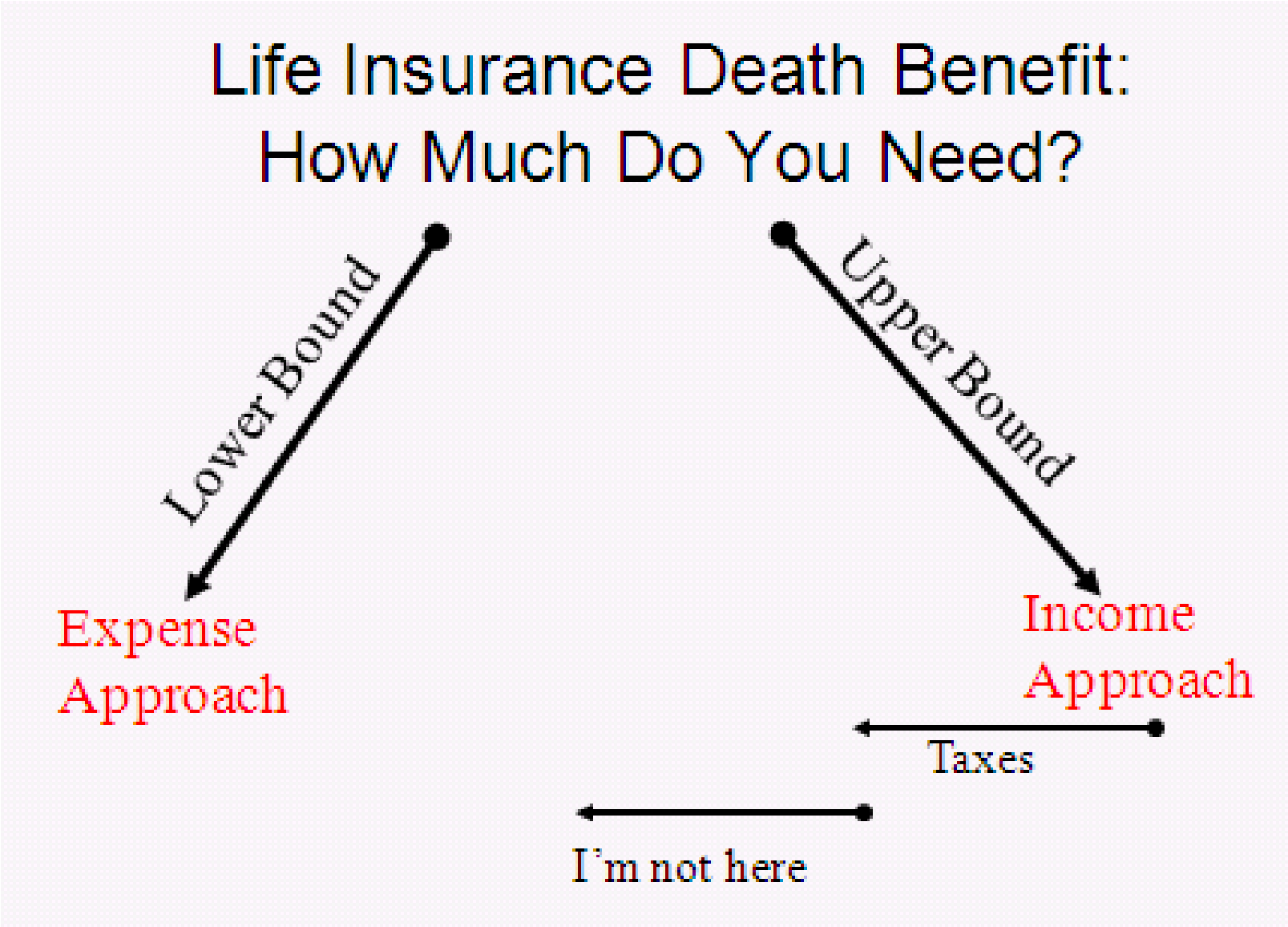
- If you have used-up RRSP and TFSA room, you might want to consider using insurance for tax-reasons as opposed to risk-management reasons (only).
- Owners of a small-business with partners (or even a family cottage) might have buy/sell clauses in their agreements, or tax liabilities upon death that might create a need for insurance.

How Much Do I Need Exactly?



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- By insuring your human capital you are implicitly **smoothing consumption** across states of nature (scenarios) in which your health is worse than expected.
- Like any other type of insurance, don't over-pay or waste premiums on unnecessary protection!