Strategic Financial Planning over the Lifecycle
Chapter #10: Mathematics of Portfolio Construction

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Ch. #10: Lecture Notes
Learning Objectives

- Understand how (why & when) diversification reduces risk.
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- Investigate the historical returns and volatility of various asset classes.
- Differentiate between manager skill (alpha) and market luck (beta).
- Learn how to construct an investment portfolio that optimally balances risk and return, taking into account your tolerance for risk.
- Finally: Integrate human capital into the asset allocation process.
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A clever broker at a large American wirehouse devised the following investment strategy in the mid-1980s.

He obtained a list of 2,000 high-net worth individuals from a local golf club association. In early January he sent a letter to (a random) 1,000 of them and suggested that they buy (i.e. go long) soybean commodity futures. At the same time he sent a different letter to the other 1,000 and suggested that they sell (i.e. go short) soybean commodity futures...

By late January soybean futures had dropped in price by 10%, so anyone from the second group who took his advice earned a 10% return in one month.

In early February he sent a follow-up letter to 500 of the “winners” from last month (i.e. those who were told to go short soybeans) and suggested that they buy the German Deutchemark. The other 500 were sent an investment recommendation letter to sell the Deutchemark.
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The Baltimore Broker: Would You Invest?

- German Deutchemark went up in February by 5%, so 500 people got letters that correctly predicted the future, two months in a row.

Anyone who had bothered taking his advice – from these 500 people – earned 5% in February and 10% in January. He then sent letters in early March to 250 of these two-time winners and suggested that they go buy SP500 stock index futures, and the other 250 were told to sell SP500 stock index futures. In March the SP500 stock index fell by about 3%, and we now have a group of 250 people who were sent correct predictions three months in a row, and had they listened to the advice earned \((1.10)(1.05)(1.03) = 1.19\)%.

He continued this process – sending contradicting predictions to the group – in April (250/2), May (125/2), June (62/2), July (31/2). By early August there are 15 people who received 7 correct predictions in a row, and had earned (on paper) about 53% over a seven month period...
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Here is a summary table of what this looks like...

<table>
<thead>
<tr>
<th>End of...</th>
<th>Investment Return</th>
<th>Probability</th>
<th>#2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 January</td>
<td>10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 February</td>
<td>5.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 March</td>
<td>4.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 April</td>
<td>7.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 May</td>
<td>5.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 June</td>
<td>8.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 July</td>
<td>6.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td><strong>53.00%</strong></td>
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<tr>
<td>January</td>
<td>10%</td>
<td>(0.5)</td>
<td>1000</td>
</tr>
<tr>
<td>February</td>
<td>5.0%</td>
<td>(0.25)</td>
<td>500</td>
</tr>
<tr>
<td>March</td>
<td>4.0%</td>
<td>(0.125)</td>
<td>250</td>
</tr>
<tr>
<td>April</td>
<td>7.0%</td>
<td>(0.0625)</td>
<td>125</td>
</tr>
<tr>
<td>May</td>
<td>5.0%</td>
<td>(0.03125)</td>
<td>62.5</td>
</tr>
<tr>
<td>June</td>
<td>8.0%</td>
<td>(0.015625)</td>
<td>31.25</td>
</tr>
<tr>
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<td>6.0%</td>
<td>(0.007813)</td>
<td>15.625</td>
</tr>
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<td><strong>53.00%</strong></td>
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- This story is a microcosm of the challenges one faces when trying to measure investment skill, and adjusting historical performance for risk.

Mutual funds companies and hedge funds have the (legal) luxury of starting many different small investment funds, with differing market views and risk levels, and then shutting down the losers and advertising the winners. This creates a variety of statistical biases in any reported claims of performance. Don't trust the numbers.

Note that there are over 200 investment companies in Canada, some offering hundreds of individual mutual funds. By one count there are over 10,000 mutual funds to choose from.

One thing is for certain: We need a precise (mathematical) framework for measuring and benchmarking investment returns, or you can fall prey to a variant of the Baltimore scam.
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The Largest Casino in the World
The Payoff on Each Table:

Bet $1

50% Chance

Pays $2

50% Chance

Pays $0
You have $10,000 which you spread (diversify) across the 10,000 roulette tables, so that you have $1 bet (invested) on each table.
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**Question:** How much money can you expect to have after all 10,000 tables stopped spinning and all the bets are settled?

**Answer:** $10,000

Reason: You can expect to get approximately 5,000 BLACK ($2) outcomes and 5,000 RED ($0) outcomes. Nothing deep or complicated, so far.
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Question #1b: What are the odds?

Question: What is the (approximate) probability that you will lose at least 50% of your money, and that the $10,000 will only be worth $5,000 (or less) at the end of all the spinning?

Answer: The odds are zero! The Law of Large Numbers (LLN) states that in the limit, the frequency of observed Black (or Red) will approach the probability of Black (or Red), which is 50/50.
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- **Hint**: For this to happen, you have to get less than 2,500 BLACK and more than 7,500 RED, in other words the ratio of RED to BLACK must be greater than 3-to-1.

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Perhaps even more surprising

Your stake of $10,000 is spread (bet) over 10,000 tables, where each table pays-out double or nothing with a 50/50 chance.

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<tr>
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<th>Investment Return</th>
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</tr>
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<tbody>
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<td>≤ $5,000</td>
<td>≤ −50%</td>
<td></td>
</tr>
<tr>
<td>≤ $7,500</td>
<td>≤ −25%</td>
<td></td>
</tr>
<tr>
<td>≤ $9,000</td>
<td>≤ −10%</td>
<td></td>
</tr>
<tr>
<td>≤ $9,500</td>
<td>≤ −5%</td>
<td></td>
</tr>
<tr>
<td>≤ $9,700</td>
<td>≤ −3%</td>
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<td>0.000%</td>
</tr>
<tr>
<td>$\leq 9,000$</td>
<td>$\leq -10%$</td>
<td>0.000%</td>
</tr>
<tr>
<td>$\leq 9,500$</td>
<td>$\leq -5%$</td>
<td>0.000%</td>
</tr>
<tr>
<td>$\leq 9,700$</td>
<td>$\leq -3%$</td>
<td>0.135%</td>
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</tbody>
</table>
Return on $10,000 Investment:
50 / 50 chance – Double or Nothing
Now Change the Table (slightly)

Favorable Bets

Bet $1

Pays $2

55% Chance

Pays $0

45% Chance
Question #2: What is expected?

- The probability of Black is 55% and Red is 45%.
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- The probability of Black is 55% and Red is 45%
- You have $10,000 which you spread (diversify) across the 10,000 roulette tables, so that you have $1 bet (invested) on each table.

You expect approximately 5,500 BLACK ($2) and 4,500 RED ($0).
The probability of Black is 55% and Red is 45%

You have $10,000 which you spread (diversify) across the 10,000 roulette tables, so that you have $1 bet (invested) on each table.

**Question**: How much money can you expect to have after all 10,000 tables stopped spinning and all the bets are settled?
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- The probability of Black is 55% and Red is 45%
- You have $10,000 which you spread (diversify) across the 10,000 roulette tables, so that you have $1 bet (invested) on each table.
- **Question**: How much money can you expect to have after all 10,000 tables stopped spinning and all the bets are settled?
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**Answer:** $11,000

You expect approximately **5,500 BLACK ($2)** and **4,500 RED ($0)**.

Now think about the same question as before. What are the odds of losing money?
Now look at the odds

Your stake of $10,000 is spread (bet) over 10,000 tables, where each table pays-out double or nothing with a 55/45 chance.

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<tr>
<td>$7,500</td>
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<td>25%</td>
</tr>
<tr>
<td>$9,000</td>
<td>$9,000</td>
<td>10%</td>
</tr>
<tr>
<td>$10,000</td>
<td>$10,000</td>
<td>0%</td>
</tr>
<tr>
<td>$10,800</td>
<td>$10,800</td>
<td>8%</td>
</tr>
</tbody>
</table>

Now you have yourself a very attractive investment...You will get at least your money back (with very high probability) and likely get a bit more, assuming you can find a Casino with such a table. Normally it is the other way around.
Return on $10,000 Investment:
55 / 45 chance – Double or Nothing
The Relative Probability
The Main Point:

- A very small edge – for example 55 to 45 ratio – can (almost) guarantee a 10% return on your $10,000 investment.
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- A very small edge – for example 55 to 45 ratio – can (almost) guarantee a 10% return on your $10,000 investment.
- But, you need (1.) a very large number of (2.) individually uncorrelated bets (tables) for this to work.
- Remember that you don't want to inadvertently place more than $1 on one table, as this reduces the diversification effect.
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- A very small edge – for example 55 to 45 ratio – can (almost) guarantee a 10% return on your $10,000 investment.
- But, you need (1.) a very large number of (2.) individually uncorrelated bets (tables) for this to work.
- Remember that you don’t want to inadvertently place more than $1 on one table, as this reduces the diversification effect.
- But what happens if you only have $100 to spread over 100 independent (tables) opportunities?
A very small edge – for example 55 to 45 ratio – can (almost) guarantee a 10% return on your $10,000 investment.

But, you need (1.) a very large number of (2.) individually uncorrelated bets (tables) for this to work.

Remember that you don’t want to inadvertently place more than $1 on one table, as this reduces the diversification effect.

But what happens if you only have $100 to spread over 100 independent (tables) opportunities?

How big is the spread in outcomes? Can you guarantee success?
Question #3: What is expected?

- You have $100 which you spread (diversify) across the 100 roulette tables, so that you have $1 bet (invested) on each table.
Question #3: What is expected?

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- **Question:** How much money can you expect to have after all 100 tables stopped spinning and all the bets are settled?
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- **Answer:** $100
You have $100 which you spread (diversify) across the 100 roulette tables, so that you have $1 bet (invested) on each table.

**Question:** How much money can you expect to have after all 100 tables stopped spinning and all the bets are settled?

**Answer:** $100

You expect **50 BLACK ($2)** and **50 RED ($0)**.
Question #3: What is expected?

- You have $100 which you spread (diversify) across the 100 roulette tables, so that you have $1 bet (invested) on each table.

**Question:** How much money can you expect to have after all 100 tables stopped spinning and all the bets are settled?

**Answer:** $100

- You expect **50 BLACK** ($2) and **50 RED** ($0).

- What are the odds of losing money?
The Odds Don’t Look as Good

Your stake of $100 is spread (bet) over 100 tables, where each table pays-out double or nothing with a 50/50 chance.

<table>
<thead>
<tr>
<th>Stake Worth…</th>
<th>Investment Return</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>$50</td>
<td>0.000%</td>
</tr>
<tr>
<td>$75</td>
<td>$75</td>
<td>0.621%</td>
</tr>
<tr>
<td>$90</td>
<td>$90</td>
<td>15.866%</td>
</tr>
<tr>
<td>$95</td>
<td>$95</td>
<td>30.854%</td>
</tr>
<tr>
<td>$99</td>
<td>$99</td>
<td>46.017%</td>
</tr>
</tbody>
</table>

Notice how 100 independent (uncorrelated) tables aren’t enough to eliminate all the risk, although you still benefit from some level of risk-reduction.
How can you compute this yourself?

- Let the random variable $w_i$ denote the outcome from the $i$'th table:

$$w_i = \begin{cases} 
2 & \text{Pr} = p \\
0 & \text{Pr} = (1 - p) 
\end{cases}$$
How can you compute this yourself?

- Let the random variable $w_i$ denote the outcome from the $i$’th table:
  
  $$w_i = \begin{cases} 
  2 & \text{Pr } = p \\
  0 & \text{Pr } = (1 - p) 
  \end{cases}$$

- Note that the **expectation** (arithmetic average) is:
  
  $$E[w_i] = 2p + 0(1 - p) = 2p$$
How can you compute this yourself?

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  \[
  w_i = \begin{cases} 
  2 & \text{Pr }= p \\
  0 & \text{Pr }= (1-p)
  \end{cases}
  \]

- Note that the **expectation** (arithmetic average) is:
  \[
  E[w_i] = 2p + 0(1-p) = 2p
  \]

- The **variance** is:
  \[
  VAR[w_i] = p(2 - 2p)^2 + (1 - p)(0 - 2p)^2 = 4p(1 - p)
  \]
How can you compute this yourself?

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- Finally, the **standard deviation** is:
  $$SD[w_i] = \sqrt{VAR[w_i]} = 2\sqrt{p(1 - p)}$$
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- Finally, the **standard deviation** is:
  
  $$SD[w_i] = \sqrt{VAR[w_i]} = 2\sqrt{p(1 - p)}$$

- For example, when $p = 0.5$ (fair table), then $E[w_i] = 1$ and $VAR[w_i] = 1$ and so $SD[w_i] = 1$. But, when $p = 0.55$, then $E[w_i] = 1.1$ and $VAR[w_i] = 4(0.55)(0.45) = 0.99$ and therefore $SD[w_i] = \sqrt{0.99} = 0.9949874$
Adding Up the Tables (i.e. Portfolio View)

Define the portfolio (total) payoff to be $W$, so that:

$$W = \sum_{j=1}^{N} w_i$$

In this case your expected portfolio payoff is:

$$E[W] = \sum_{j=1}^{N} E[w_i]$$

The variance of the portfolio payoff is:

$$\text{VAR}[W] = \sum_{j=1}^{N} \text{VAR}[w_i]$$

This equation is true only when the correlation between the individual tables (returns) is zero. Otherwise you have to add a covariance (a.k.a. correlation) term which will increase or reduce the portfolio variance, depending on its sign.
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- Define the portfolio (total) payoff to be $W$, so that:

$$W = \sum_{j=1}^{N} w_i$$

- In this case your **expected** portfolio payoff is:

$$E[W] = \sum_{j=1}^{N} E[w_i] = N2p$$

The variance of the portfolio payoff is:

$$\text{VAR}[W] = \sum_{j=1}^{N} \text{VAR}[w_i] = N4p(1-p)$$

This equation is true only when the correlation between the individual tables (returns) is zero. Otherwise you have to add a covariance (a.k.a. correlation) term which will increase or reduce the portfolio variance, depending on its sign.
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What can I do with these formulas?

- Assume that you have $250 spread across 250 tables, where $p = 0.53$ (BLACK) and $(1 - p) = 0.47$ (RED).

  Question: What is the probability you will lose 10% (or more), and you end-up $225 (or less) at the end of the 250 spins?

  Answer: The expected outcome is:

  $E[W] = 250 \cdot 0.53 = 265$,

  which is an expected investment return of

  \[
  \frac{265 - 250}{250} = 6\%.
  \]

  The variance of the total payout is:

  $VAR[W] = 250 \cdot 0.53 \cdot 0.47 = 249.1$.

  The standard deviation of the total payout is:

  $SD[W] = \sqrt{249.1} = 15.7829$,

  which is $15.7829 / 250 = 6.31\%$ of the investment.
What can I do with these formulas?

- Assume that you have $250 spread across 250 tables, where \( p = 0.53 \) (BLACK) and \( (1 - p) = 0.47 \) (RED).

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What can I do with these formulas?

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- **Question**: What is the probability you will lose 10% (or more), and you end-up $225 (or less) at the end of the 250 spins?
- **Answer**: The expected outcome is:

\[
E[W] = 250 \times 2 \times 0.53 = 265,
\]

which is an **expected investment return** of \((265 - 250)/250 = 6\%\)
Assume that you have $250 spread across 250 tables, where \( p = 0.53 \) (BLACK) and \( (1 - p) = 0.47 \) (RED).

**Question:** What is the probability you will lose 10% (or more), and you end-up $225 (or less) at the end of the 250 spins?

**Answer:** The expected outcome is:

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E[W] = 250 \times 2 \times 0.53 = 265,
\]

which is an **expected investment return** of \((265 - 250)/250 = 6\%\).

The variance of the total payout is:

\[
VAR[W] = 250 \times 4 \times (0.53)(0.47) = 249.1
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What can I do with these formulas?

- Assume that you have $250 spread across 250 tables, where \( p = 0.53 \) (BLACK) and \( (1 - p) = 0.47 \) (RED).
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  \[
  SD[W] = \sqrt{249.1} = 15.7829,
  \]

  which is \(15.7829/250 = 6.31\%\) of the investment.
Use the Central Limit Theorem (CLT)

When you add-up a large number \( (N > 30) \) of independent random variables \( (w_i) \) the resulting r.v. \( (W) \) will be normally distributed with mean \( E[W] \) and standard deviation \( SD[W] \).
Finally, a Number

- The probability of ending-up with \( Y \) or less is written formally as:

\[
\Pr[W \leq Y] = \Pr\left[ \frac{W - E[W]}{SD[W]} \leq \frac{Y - E[W]}{SD[W]} \right].
\]

As a result of the CLT we can write this as:

\[
\Pr[W \leq Y] = \Pr[Z \leq \frac{225 - 250}{0.53}].
\]

In our case we substitute \( Y = 225, N = 250, p = 0.53 \):

\[
\Pr[W \leq 175] = \Pr[Z \leq 2.5344].
\]

Finally, we take the z-score to the normal tables to obtain \( \Pr[Z < 2.5344] = 0.563\% \). You can get this in Excel using the `NORMSDIST(-2.5344)` function.
The probability of ending-up with $Y$ or less is written formally as:

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As a result of the CLT we can write this as:

$$\Pr[W \leq Y] = \Pr \left[ Z \leq \frac{Y - N2p}{2\sqrt{Np(1-p)}} \right],$$

where the symbol $Z$ denotes a standard (mean zero, variance 1) normal random variable.
Finally, a Number

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- In our case we substitute $Y = 225$, $N = 250$, $p = 0.53$:

$$\Pr[W \leq 175] = \Pr[Z \leq \frac{225 - 265}{2\sqrt{250(1-0.53)}}] = \Pr[Z \leq -2.5344].$$
Finally, a Number

- The probability of ending-up with $Y$ or less is written formally as:
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  \[ \Pr[W \leq 175] = \Pr[Z \leq \frac{225 - 265}{15.7829}] = \Pr[Z \leq -2.5344]. \]

- Finally, we take the z-score to the normal tables to obtain
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The probability of ending-up with $Y$ or less is written formally as:

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You can get this in Excel using the $NORMSDIST(-2.5344)$ function.
Practice Questions

**Q1:** You have $500 which you have allocated evenly across 500 tables, in which $p = 0.53$. What is the probability you end-up with less than $500 at the end of the round?
Practice Questions

- **Q1**: You have $500 which you have allocated evenly across 500 tables, in which $p = 0.53$. What is the probability you end-up with less than $500 at the end of the round?

- **Q2**: You have $500 which you have allocated evenly across 50 tables, in which $p = 0.53$. What is the probability you end-up with less than $500 at the end of the round?
Practice Questions

Q1: You have $500 which you have allocated evenly across 500 tables, in which $p = 0.53$. What is the probability you end-up with less than $500 at the end of the round?

Q2: You have $500 which you have allocated evenly across 50 tables, in which $p = 0.53$. What is the probability you end-up with less than $500 at the end of the round?

Q3: How many independent tables ($N$) do you need to split the $500 across, so that the probability of losing money (with $p = 0.53$) is no more than 15%?
<table>
<thead>
<tr>
<th>N: Tables</th>
<th>297</th>
<th>500</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Capital</td>
<td>$500.0</td>
<td>$500.0</td>
<td>$500.0</td>
</tr>
<tr>
<td>Bet per Table (BPT)</td>
<td>$1.68</td>
<td>$1.00</td>
<td>$10.0</td>
</tr>
<tr>
<td>Pr [Black]</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>E[W]</td>
<td>$530.0</td>
<td>$530.0</td>
<td>$530.0</td>
</tr>
<tr>
<td>VAR[W]</td>
<td>$838.72</td>
<td>$498.20</td>
<td>$4,982.0</td>
</tr>
<tr>
<td>SD[W]</td>
<td>$28.961</td>
<td>$22.320</td>
<td>$70.583</td>
</tr>
<tr>
<td>Z-score: (Y-E[W])/SD[W]</td>
<td>-1.036</td>
<td>-1.344</td>
<td>-0.425</td>
</tr>
<tr>
<td>Probability</td>
<td>15.01%</td>
<td>8.95%</td>
<td>33.54%</td>
</tr>
</tbody>
</table>

**Answer to Practice Questions**

**Initial Capital**
- 297Table: $500.0
- 500Table: $500.0
- 50Table: $500.0

**Bet per Table (BPT)**
- 297Table: $1.68
- 500Table: $1.00
- 50Table: $10.0

**Pr [Black]**
- 297Table: 0.53
- 500Table: 0.53
- 50Table: 0.53

**E[W]**
- 297Table: $530.0
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**VAR[W]**
- 297Table: $838.72
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- 50Table: $4,982.0

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**Probability**
- 297Table: 15.01%
- 500Table: 8.95%
- 50Table: 33.54%
Real World Correlations are Rarely Zero

The Lingo of Portfolio Theory

Portfolio Risk - Std. Dev.

Number of Stocks in Portfolio

Total risk

Company-specific risk

Diversifiable risk

Unsystematic risk

Market Risk

Non-diversifiable or Systematic risk

CHM (Cambridge 2012) Ch. #10: Lecture Notes
## Diversification in Practice: Asset Class Returns

<table>
<thead>
<tr>
<th>Year</th>
<th>U.S. Bonds</th>
<th>U.S. Stocks</th>
<th>Emerging Markets</th>
<th>Real Estate</th>
<th>Diversified Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
<td></td>
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<tr>
<td>2007</td>
<td></td>
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<tr>
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<tr>
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<tr>
<td>2003</td>
<td></td>
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</tr>
<tr>
<td>2002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Source:** Morningstar as of 12/31/2009 (U.S. Based)
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<table>
<thead>
<tr>
<th>Year</th>
<th>U.S. Bonds</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>5.93%</td>
<td>28.34%</td>
<td>74.50%</td>
<td>27.77%</td>
<td>18.91%</td>
</tr>
<tr>
<td>2008</td>
<td>5.24%</td>
<td>-37.31%</td>
<td>-53.33%</td>
<td>-49.87%</td>
<td>-35.65%</td>
</tr>
<tr>
<td>2007</td>
<td>6.97%</td>
<td>5.14%</td>
<td>39.39%</td>
<td>-7.69%</td>
<td>16.23%</td>
</tr>
<tr>
<td>2006</td>
<td>4.33%</td>
<td>15.72%</td>
<td>32.17%</td>
<td>36.44%</td>
<td>2.07%</td>
</tr>
<tr>
<td>2005</td>
<td>2.43%</td>
<td>6.12%</td>
<td>34.00%</td>
<td>11.41%</td>
<td>21.36%</td>
</tr>
<tr>
<td>2004</td>
<td>4.43%</td>
<td>11.95%</td>
<td>25.55%</td>
<td>31.43%</td>
<td>9.15%</td>
</tr>
<tr>
<td>2003</td>
<td>4.10%</td>
<td>31.06%</td>
<td>55.82%</td>
<td>31.71%</td>
<td>23.93%</td>
</tr>
<tr>
<td>2002</td>
<td>10.25%</td>
<td>-21.54%</td>
<td>-6.17%</td>
<td>-9.24%</td>
<td>25.91%</td>
</tr>
</tbody>
</table>

**Source:** Morningstar as of 12/31/2009 (U.S. Based)
The spreadsheet (on the CMD) displays the historical (monthly) returns from various equity asset classes over the last few decades.
Historical Returns from Equity

- The spreadsheet (on the CMD) displays the historical (monthly) returns from various equity asset classes over the last few decades.
- **Q1:** What was the expected return, standard deviation and correlation of the various equity asset classes?
The spreadsheet (on the CMD) displays the historical (monthly) returns from various equity asset classes over the last few decades.

Q1: What was the expected return, standard deviation and correlation of the various equity asset classes?

Q2: What is the standard deviation of a portfolio that is 50% in Asset Class A and 50% in Asset Class B?
In contrast to the (simple) calculations used earlier, when there is a non-zero correlation (i.e. dependence) between assets in a portfolio, the variance can’t be obtained by adding the individual variances.

Let $\sigma_A$ denote the standard deviation of asset class $A$ and let $\sigma_B$ denote the standard deviation of asset class $B$, and let $\rho_{AB}$ denote the correlation:

In this case $\sigma = \sqrt{f_A^2 \sigma_A^2 + f_B^2 \sigma_B^2 + 2f_A f_B \rho_{AB} \sigma_A \sigma_B}$, where $f_A$ denotes the fraction of the portfolio invested in asset class $A$, and $f_B$ denotes the fraction invested in asset class $B$. Don’t be alarmed... Examples are on the way.
In contrast to the (simple) calculations used earlier, when there is a non-zero correlation (i.e. dependence) between assets in a portfolio, the variance can’t be obtained by adding the individual variances.

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where $f_A$ denotes the fraction of the portfolio invested in asset class A, and $f_B$ denotes the fraction invested in asset class B.

Don’t be alarmed...Examples are on the way.
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Let $\sigma_A$ denote the standard deviation of asset class $A$ and let $\sigma_B$ denote the standard deviation of asset class $B$, and let $\rho_{AB}$ denote the correlation:

In this case

$$\sigma = \sqrt{f_A^2 \times \sigma_A^2 + f_B^2 \times \sigma_B^2 + 2 \times f_A \times f_B \times \rho_{AB} \times \sigma_A \times \sigma_B},$$

where $f_A$ denotes the fraction of the portfolio invested in asset class $A$, and $f_B$ denotes the fraction invested in asset class $B$. 

Don’t be alarmed...Examples are on the way.
In contrast to the (simple) calculations used earlier, when there is a non-zero correlation (i.e. dependence) between assets in a portfolio, the variance can’t be obtained by adding the individual variances.

Let $\sigma_A$ denote the standard deviation of asset class $A$ and let $\sigma_B$ denote the standard deviation of asset class $B$, and let $\rho_{AB}$ denote the correlation:

In this case

$$\sigma = \sqrt{f_A^2 \times \sigma_A^2 + f_B^2 \times \sigma_B^2 + 2 \times f_A \times f_B \times \rho_{AB} \times \sigma_A \times \sigma_B},$$

where $f_A$ denotes the fraction of the portfolio invested in asset class $A$, and $f_B$ denotes the fraction invested in asset class $B$.

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Numerical Example

- Assume that you have 100% of your wealth (money) invested in asset class A, with a measured (historical) standard deviation on $\sigma_A = 40\%$ (which is quite high)
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Question: What is the portfolio standard deviation (\( \sigma \)) if you allocate 20%, 50%, and 80% to this new asset class B?

Hint: Use the formula with \( f_B = 0.20 \), \( f_A = 0.80 \), \( f_B = 0.50 \), and \( f_A = 0.50 \).

Results: See the table.
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### Portfolio Analysis: Correlation $\rho_{AB} = +25\%$

<table>
<thead>
<tr>
<th>Asset B</th>
<th>$\sigma_B$</th>
<th>Asset A</th>
<th>$\sigma_A$</th>
<th>Portfolio Risk ((\sigma))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>25%</td>
<td>100%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>20%</td>
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<td></td>
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<tr>
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<tr>
<td>100%</td>
<td>25%</td>
<td>0%</td>
<td>40%</td>
<td></td>
</tr>
</tbody>
</table>
### Numerical Results

**Portfolio Analysis: Correlation** \( \rho_{AB} = +25\% \)

<table>
<thead>
<tr>
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<th>( \sigma_B )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>25%</td>
<td>100%</td>
<td>40%</td>
<td>40.00%</td>
</tr>
<tr>
<td>20%</td>
<td>25%</td>
<td>80%</td>
<td>40%</td>
<td>33.60%</td>
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<td>60%</td>
<td>40%</td>
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<tr>
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<td>50%</td>
<td>40%</td>
<td>26.10%</td>
</tr>
<tr>
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<td>25%</td>
<td>40%</td>
<td>40%</td>
<td>24.52%</td>
</tr>
<tr>
<td>70%</td>
<td>25%</td>
<td>30%</td>
<td>40%</td>
<td>23.56%</td>
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<td>40%</td>
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<td>100%</td>
<td>25%</td>
<td>0%</td>
<td>40%</td>
<td>25.00%</td>
</tr>
</tbody>
</table>
Relationship Between Risk and Return

- Depending on your attitude towards financial risk (a.k.a. risk aversion) which we explored in the discussion of insurance, you will allocate more (or less) to the riskier asset class.

For investors that are risk tolerant, they will allocate themselves higher on the efficient frontier. On the other hand, if you are more risk averse you will allocate yourself lower on the efficient frontier.

There is no universally optimal (right) portfolio for everyone...the mix depends on your preferences.
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The Same Idea Can Be Applied to Human Capital

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**Question:** Given a particular job risk-profile what is the optimal allocation to stocks for a given risk tolerance?
<table>
<thead>
<tr>
<th>Age...</th>
<th>Tenured</th>
<th>Bankruptcy</th>
<th>Mech.</th>
<th>Invest.</th>
</tr>
</thead>
<tbody>
<tr>
<td>45: Stock %</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insurance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55: Stock %</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Note:** These are idealized values based on a model that only varies the correlation ($\rho$) between wages and stock market returns, as well as an average relative risk risk aversion ($\gamma = 2$).
### Human Capital Asset Allocation Model (HCAM)

You (Expect to) Earn $100K per Year

<table>
<thead>
<tr>
<th>Age...</th>
<th>Tenured</th>
<th>Bankruptcy</th>
<th>Mech.</th>
<th>Invest.</th>
</tr>
</thead>
<tbody>
<tr>
<td>45: Stock %</td>
<td>280%</td>
<td>170%</td>
<td>125%</td>
<td>60%</td>
</tr>
<tr>
<td>Insurance</td>
<td>$1.9M</td>
<td>$1.5M</td>
<td>$1.4M</td>
<td>$1.3M</td>
</tr>
<tr>
<td>55: Stock %</td>
<td>85%</td>
<td>70%</td>
<td>50%</td>
<td>35%</td>
</tr>
<tr>
<td>Insurance</td>
<td>$0.8M</td>
<td>$0.6M</td>
<td>$0.5M</td>
<td>$0.4M</td>
</tr>
</tbody>
</table>

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- The **Risk of the Sum** is less than the **Sum of the Risks**: (RS < SR)
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